Appendix 1: Mathematical specification of the model

Model notation:

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t half a year (*t* = 1, 2, …, *T*)

z zone/region (*z* = 1, 2, …, *Z*), where *Z* = 1 indicates no adoption of zoning

s beef type or cattle type (*s* = *m*, *c*), where *m* indicates beef from young steers and young female cattle raised for fattening and *c* indicates beef from females used for breeding.

a age cohort ($a = 1, 2, ..., A_s$), where $A_m = 10$ for *s* = *m* and $A_c = 20$ for *s* = *c*

k beef market ($k = e_f, e_e, d_1, d_2, ..., d_Z$), where e indicates export market (e_f: FMD free export market; e_e: FMD endemic export market) and d indicates domestic markets (total *Z* domestic markets).

Variables

Note: All variables are time dependent and represent values at the start of a time period or during a time period.

$p_s(t,z)$ price of beef of type *s* supplied from zone *z*, fob major port

$p_s^k(t,z)$ price of beef of type *s* on market *k* supplied from zone *z,* fob major port

$q_s^k(t,z)$ quantity of beef of type *s* on market *k* supplied from zone *z*

$sl_a^s(t,z)$ number of cattle of type *s* and age group *a* slaughtered at zone *z*

$x_a^s(t,z)$ number of cattle of type *s* and age group *a* at zone *z*

$xs(t,z)$ number of female calves born during period *t* diverted to fattening in that time period at zone *z*

$l_a^s(t,z)$ price of cattle of type *s* and age group *a* at zone *z*

$ssl_a^s(t,z)$ rent to slaughter of cattle of type *s* and age group *a* at zone *z*

$sxs(t,z)$ rent to female calves born during period *t* diverted to fattening in that time period at zone *z*

Parameters

$a_a(z)$ proportion of calves marked per breeding female at zone *z*

$B_s^k(z)$ intercept parameter in linear inverse demand function for beef of type *s* supplied from zone *z* on market *k*

$C_s^k(z)$ slope parameter in linear inverse demand function for beef of type *s* supplied from zone *z* on market *k*

cfe_a^s half yearly costs per head of cattle of type *s* and age group *a* for each zone*,* ie feeding costs and all other yearly costs to farm gate

$csl_a^s(z)$ cost per head of cattle of type *s* and age group *a* at zone *z* from farm gate to conversion into beef fob major port, ie slaughter costs and other costs excluding transportation costs.

m_a^s proportion of cattle of type *s* and age group *a* dying half yearly from natural causes

r half yearly rate of discount

w_a^s saleable beef weight per head of cattle slaughtered of type *s* and age group *a*

$t^{k}(z)$ transportation cost of per kilogram of beef from zone *z* to market *k*

Bioeconomic model of Australian beef production, consumption and export trade

The assumptions about the biological and economic conditions of the Australian cattle industry that make up the model are represented as follows: for $t = 1,2,\Lambda\ ,T,$ and $z = 1,2,\Lambda\ ,Z$ [1]

1 - $$x_1^m(t+1,z) = \tfrac{1}{2}\sum_{a=1}^{A_c} a_a(z)x_a^c(t-1,z)/2 + xs(t+1,z),$$ [2]

for each time step, the number of cattle of age one for nonbreeding purposes equals the number of male calves and female calves diverted from breeding. It is assumed that male calves and female calves are equal in number.

2 - $$x_1^c(t+1,z) = \tfrac{1}{2}\sum_{a=1}^{A_c} a_a(z)x_a^c(t-1,z)/2 - xs(t+1,z),$$

for each time step, the number of breeding females of age one equals the number of female calves less the number diverted to nonbreeding purposes.

3 - $$xs(t+1,z) \le \tfrac{1}{2}\sum_{a=1}^{A_c} a_a(z)x_a^c(t-1,z)/2,$$

for each time step, the number of female calves diverted to nonbreeding purposes cannot exceed the number of female calves born.

4 - $$x_{a+1}^s(t+1,z) = (1-m_a^s)x_a^s(t,z) - sl_a^s(t,z) \text{ for } a = 1,2,\ldots,A_s - 1 \text{ and}$$

$$x_{A_s}^s(t+1,z) = (1-m_{A_s-1}^s)x_{A_s-1}^s(t,z) - sl_{A_s-1}^s(t,z) + (1-m_{A_s}^s)x_{A_s}^s(t,z) - sl_{A_s}^s(t,z), \text{ all for } s = c,m$$ [3]

for each time step, the number of cattle of each type of any age equals the number of that type of cattle that were one age younger the previous time step less a proportional natural mortality and less the number slaughtered.

5 - $$sl_a^s(t,z) \le (1-m_a^s)x_a^s(t,z) \text{ for } s = c,m \text{ and } a = 1,2,\ldots,A_s,$$

for each time step, the maximum slaughter of cattle of each age and type cannot exceed the number of surviving cattle of that age and type.

$$p_s^k(t) = B_s^k + C_s^k \sum_z q_s^k(t,z), \text{ for } s = c,m,\ k = d_1,d_2,\Lambda\ ,d_Z \text{ (ie domestic markets);}$$

6 - $$p_s^k(t,z) = B_s^k(z) + C_s^k(z)q_s^k(t,z), \text{ for } s = c,m,\ k = e_f, e_e \text{ (ie export markets)},$$

for each time step, the volume demanded of beef of any type on any market depends on the price of that beef according to a downward sloping linear demand function with parameters that are constant over time. Note that prices of beef in the domestic markets are assumed independent of the origin of beef, while the prices in the export markets are assumed dependent on the origin of beef.

[1] In the case of no zoning, $Z = 1$ (ie one single zone for the whole of Australia).
[2] Breeding cows produce calves two time steps (ie one year) later. It is assumed that at each time step only half of the cows breed in order to avoid cows breeding twice per year.
[3] The oldest age groups for breeding and nonbreeding cattle are heterogeneous groups containing all animals beyond certain ages. Slightly modified conditions apply for these age groups. However, these terminal age groups are chosen so as to be old enough never to have any surviving animals in them for whatever event or policy simulation.

7 - $$\sum_{k=e_f,e_e,d_1,\ldots,d_Z} q_s^k\ (t,z) \le \sum_{a=1}^{A_s} sl_a^s(t,z)w_a^s\text{, for } s = c, m\text{,}$$

when there is no closure of any domestic or export market, then the total quantity of beef sold of a type on all markets cannot exceed the total quantity of that type produced from slaughter.

When there is a foot and mouth disease outbreak in Australia, then

- for the FMD affected zone [4], ie $z = z_{FMD}$,

$$\sum_{s=c,m}\ \sum_{k=e_f,e_e,d_1,\ldots,d_Z} q_s^k\ (t,z_{FMD}) \le \sum_{s=c,m}\ \sum_{a=1}^{A_s} sl_a^s(t,z_{FMD})w_a^s \text{ and}$$

$q_s^{e_f}(t,z_{FMD}) = 0$ for $s = c, m$ and

$q_s^k(t,z_{FMD}) = 0$ for $s = c, m$, and $k = d_z$ for all $z \ne z_{FMD}$

all for $t = 1, T_1$;

- for the unaffected zones, ie $z \ne z_{FMD}$,

$$\sum_{s=c,m}\ \sum_{k=e_f,e_e,d_1,\ldots,d_Z} q_s^k\ (t,z) \le \sum_{s=c,m}\ \sum_{a=1}^{A_s} sl_a^s(t,z)w_a^s \text{ and}$$

$q_s^{e_f}(t,z) = 0$ for $s = c, m$

all for $t = 1, T_2$ $(T_2 \le T_1)$, [5]

when the FMD free export market for all types of beef from the affected region in Australia is closed for T_1 periods and from the unaffected region for T_2 periods due to FMD, and when the unaffected domestic markets for all types of beef from the FMD affected region are also closed for T_1 periods, then for each region the total quantity of beef sold on the remaining markets cannot exceed the total quantity of beef produced from slaughtering regardless of types. After T_1 or T_2 periods, the relevant markets are assumed to reopen.

8 - $$p_s(t,z)w_a^s - csl_a^s(z) - (1+r)^{-1}\ l_{a+1}^s(t+1,z) - s\,sl_a^s(t,z) \le 0\text{, for } a = 1,2,\ldots,A_s - 1$$

and $$p_s(t,z)w_{A_s}^s - csl_{A_s}^s(z) - (1+r)^{-1}\ l_{A_s}^s(t+1,z) - s\,sl_{A_s}^s(t,z) \le 0\text{, for } s = c, m\text{,}$$ [2]

for each time step, the revenue from slaughter of a head of cattle of each age or type cannot exceed the sum of the slaughter cost per head, the present value of a one period older animal of the next time step and any slaughter rent. For the period of closure of the FMD free export market for all types of beef due to FMD, there will be one common price for both beef types and the prices $p_s(t,z)$ should be replaced by $p(t,z)$ for the corresponding regions.

[4] For the case of no zoning, the FMD affected zone means the whole of Australia.
[5] This is just one of the scenarios for the duration of market closure. Other scenarios for the duration of market closure can be specified similarly.

9 - $l_1^m(t+1,z) - l_1^c(t+1,z) - sxs(t+1,z) \leq 0$, for $t = 1,2,...,T$,

for each time step, the present value of the excess of next time step's value of a one age old nonbreeding animal over that of a breeding animal cannot exceed any rent to female calves.

10 - $l_a^s(t,z) \geq (1+r)^{-1} l_{a+1}^s(t+1,z)(1-m_a^s) + ssl_a^s(t,z)(1-m_a^s) - cfe_a^s$

$+ \frac{a_a(z)}{4}\{(1+r)^{-2}[l_1^c(t+2,z) + l_1^m(t+2,z) + sxs(t+2,z)]\}$, for $a = 1,2,...,A_s - 1$ and

$l_{A_s}^s(t,z) \geq (1+r)^{-1} l_{A_s}^s(t+1,z)(1-m_{A_s}^s) + ssl_{A_s}^s(t,z)(1-m_{A_s}^s) - cfe_{A_s}^s$

$+ \frac{a_{A_c}(z)}{4}\{(1+r)^{-2}[l_1^c(t+2,z) + l_1^m(t+2,z) + sxs(t+2,z)]\}$, all for $s = c, m$,

for each time step, the value of an animal of any age and type cannot be less than the present value at the next time step of a surviving one period older animal of that type plus any slaughter rent minus annual feeding cost plus, for breeding cattle only, the present value of offspring at age one two time steps later.

11 - $p_s^k(t) - t^k(z) - p_s(t,z) \leq 0$, for $s = c, m$ and $k = d_1, d_2, \Lambda, d_Z$ (ie domestic markets) ;

$p_s^k(t,z) - t^k(z) - p_s(t,z) \leq 0$, for $s = c, m$ and $k = e_f, e_e$ (ie export markets) ,

for each time step, the price of beef of type *s* on any market *k* cannot exceed the price of that type of beef plus the transportation cost. When there is market closure due to a FMD for a number of periods, all types of beef will be sold on the remaining markets at a single price, $p(t,z)$, regardless of types. For those periods with market closure the following conditions would apply:

- for $z = z_{FMD}$,

  for $k = d_{Z_{FMD}}$ (ie domestic market), $p_s^k(t) - t^k(z_{FMD}) - p(t, z_{FMD}) \leq 0$ and

  for $k = e_e$ (ie export market), $p_s^k(t, z_{FMD}) - t^k(z_{FMD}) - p(t, z_{FMD}) \leq 0$,

  all for $s = c, m$ and $t = 1,...,T_1$;

- for $z \neq z_{FMD}$,

  for any domestic market, $p_s^k(t) - t^k(z) - p(t,z) \leq 0$ and

  for $k = e_e$ (ie export market), $p_s^k(t,z) - t^k(z) - p(t,z) \leq 0$,

  all for $s = c, m$ and $t = 1,...,T_2$.

The problem is to find the model solution, ie the values of all variables in the model that satisfy the conditions (1)–(11) and for which pure profits for each time step and thus over the whole time horizon are maximised and equal to zero. This guarantees that no market participant is in a position to gain from changing any decision from the value in the model solution.

For this problem, the numbers in the cattle herd by age and type for the first time step, $x_a^s(1,z)$, are given. Also, the numbers of breeding cows by age immediately before the first time step, $x_a^c(0,z)$, are given. However, there are multiple possible end conditions. For example, terminal cattle numbers or terminal cattle prices could be prescribed.

The approach that is followed here is to assume that the cattle industry is currently in the (optimal) steady state of the above system. In the absence of any disturbance, the system would remain in that steady state, and $x_a^s(t,z)=x_a^s(z),\ l_a^s(t,z)=l_a^s(z)$ [6], for $t=1,2,\ldots,T+1$. For all policy or event simulations to be reviewed, the prescribed end-state is the same (optimal) steady state. The time horizon is chosen far enough out to allow the system to reach this prescribed end-state.

The pure profit criterion, C, to be maximised over the time horizon is

$$C=\sum_{t=1}^{T}(1+r)^{-t}\{\sum_{z}\sum_{s}\sum_{k}(p_s^k(t,z)-t^k(z))q_s^k(t,z)-\sum_{z}\sum_{s}\sum_{a}csl_a^s(z)sl_a^s(t,z)$$
$$-\sum_{z}\sum_{s}\sum_{a}cfe_a^s x_a^s(t,z)\ \}$$
$$+(1+r)^{-(T+1)}\sum_{z}\sum_{s}\sum_{a}l_a^s(T+1,z)x_a^s(T+1,z)-(1+r)^{-1}\sum_{z}\sum_{s}\sum_{a}l_a^s(1,z)x_a^s(1,z)$$
$$+(1+r)^{-(T+2)}\sum_{z}\sum_{s}l_1^s(T+2,z)x_1^s(T+2,z)-(1+r)^{-2}\sum_{z}\sum_{s}l_1^s(2,z)x_1^s(2,z).$$

This criterion is the sum of discounted revenue minus slaughter costs minus feeding costs over all time steps of the time horizon plus the increase in the present value of the livestock from time step 1 to time step *T*+1, plus the increase in the present value of the calves born at time step 2 from the breeding cows before the time step started and the calves to be born at the time step *T*+2 from the breeding cows at the time step *T*.

When there is market closure for a number of periods due to FMD industry participants will alter their decisions. From the initial state cattle numbers, $x_a^s(1,z)=x_a^s(z)$, an adjustment path will be followed that will extend beyond the reopening of the corresponding markets at the corresponding period (T_1+1 or T_2+1).

The cost to Australia of the FMD induced export market closure is estimated as the net present value of losses and gains to Australian producers and consumers for the time of the deviation of the adjustment path from the (optimal) steady state path.

[6] Given the time step is half a year, if it is considered that numbers in the herds differ in the first half year from those in the second half year, then a periodic steady state can be specified as follows:

$$x_a^s(t+2,z)=x_a^s(t,z)=x1_a^s(z);\ x_a^s(t+3,z)=x_a^s(t+1,z)=x2_a^s(z);$$
$$l_a^s(t+2,z)=l_a^s(t,z)=l1_a^s(z);\ l_a^s(t+3,z)=l_a^s(t+1,z)=l2_a^s(z),$$
for $t=1,2,\ldots,T-3$.