Structuring Exotic Options Contracts on Water to Improve the Efficiency of Resource Allocation in the Australian Water Market

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Abstract

The potential economic benefits that options contracts bring to the Murray Valley water market in Australia are assessed. Exotic call option prices are estimated using Black-Scholes and skewness-and-kurtosis-amended Black-Scholes option closed-form pricing methods that are based on mean weekly water prices between 2004 and 2008. While options would result in significant economic benefits through more efficient trade of water on the open market for lower-value crops, there were mixed results from attempts to price them. Results show that use of the standard Black-Scholes formula is likely to undervalue option prices considerably at all but improbably low levels of volatility in water prices. Water option prices are high relative to the net present value of option benefits for recent levels of volatility, which is likely to discourage the development of a water options market. Alternatives to reduce the option prices are discussed. Other potential constraints to the implementation of a water options trading system are outlined.

Key words: options, skewness-and-kurtosis-amended Black-Scholes model, volatility, water.

1. Introduction

Farmers in Eastern Australia have experienced favourable seasons in recent years and memories of the ordeals of the long drought of the early to mid-2000s have receded. But the prospect of another major drought occurring will no doubt still be in the minds of most farmers, and farms in a number of areas are already appearing to enter into drought conditions. Although the Murray River still has good water levels, and there are no immediate threats of shortages, the best time to consider measures to manage drought is when conditions remain satisfactory and before the next drought occurs.

Water has long been recognised as a scarce resource in inland Australia. It is now widely acknowledged that its efficient allocation is important to allow agriculture, urban populations and the natural environment to co-exist. The scarce and uncertain availability of water has been highlighted in recent times by the drought that has afflicted South-Eastern Australia and significantly reduced the recharge of water stocks. Together with a population growth rate faster than the growth in water supply facilities, it has created an imbalance between supply and demand that has resulted in large increases in water prices (WSAA 2005).
The relative scarcity of water in the Australian landscape has resulted in water playing a key role in public policy making over a long period. This role has been especially evident since the turn of the century with the launch of the National Water Initiative and the $10 billion ‘Ten Point Plan’ by the Howard coalition government in 2007. The aim of the ‘Ten Point Plan’ was to upgrade water infrastructure and technology and to address the over-allocation of water in the Murray Darling Basin (Sim 2007). The newly elected coalition government has vowed to return to this water strategy.

Prior to these schemes, the most important water policy change in recent times was the decision by the Council of Australian Governments (COAG) in 1994 to implement a water reform process. This reform included the imposition of the Murray-Darling Cap on Surface Water Diversion (Alexandra 2008). State governments introduced a system of water entitlements or allocations backed by the separation of water property rights from land title. This reform provided clear specification of entitlements in terms of ownership, volume, reliability, transferability and, if applicable, quality (Bjornlund and O’Callaghan 2003). By using a free market as the price discovery tool for water, the interaction of supply and demand theoretically determines what water is worth and then directs that water to its highest-value enterprise.

The introduction of an openly traded water market in Australia has allowed this reallocation to happen to some extent, but the continuous intervention by the various government agencies in this market limits its efficiency (Martin, Williams and Stone 2008). A possible solution is the use of derivative securities, namely options contracts, in the water market. The underlying idea of an options contract is to create a risk-sharing mechanism that, ex ante, provides a ‘fair price’ to both parties to the contract and ensures that the transfer of risk between the two parties is mutually beneficial.

Water options contracts also provide a means for governments at the federal and state levels to improve their decision making in respect of water policies in a risky environment. Quiggin (2001) reviewed the negative environmental consequences of agricultural development in Australia, focusing particularly on the problems of salinisation, waterlogging, algal blooms, loss of biodiversity and loss of habitat in the Murray–Darling Basin. He argued that ‘the variability of flows has encouraged over-allocation of irrigation water leading to problems of unreliable supplies, low residual flows and conflict between upstream and downstream water users’ (Quiggin 2001, p. 68). The use of options could help overcome these problems by enabling policy makers to expand the range of their alternative future choices. Given that river flows vary over time and are highly uncertain, the Federal and State Governments could trade in options through their agencies to allocate future flows in a more socially efficient way in competition with irrigation and urban water users. The Murray Darling Basin Commission (MDBC) undertook its first purchase of irrigation water to provide environmental flows to the wetlands at Narran Lakes to sustain breeding birds in the 2007-2008 summer (Alexandra 2008).

The aim in the first part of this paper is to determine whether this mutually beneficial transfer of risk between parties can result in increased efficiency in the allocation of water, which will be measured by the level of net economic benefit these contracts generate. In the second part of the study, an attempt is made to price a call options contract on water using a financial options pricing model. This is illustrated through a case study covering a recent period (August 2004 to November 2008) that encompassed almost two years of relatively favourable water supplies with low spot prices and two and a half years of drought with high to exceptionally high spot prices. It is based on the temporary and permanent trade of water entitlements in the Murray River Regulated System in South-Eastern Australia. This system was selected because it has the largest volume of water traded on the open market (NSW DNR 2008). It is the most developed of the Australian water markets and most likely to benefit from the inclusion of options contracts in the marketplace. Call option prices are then compared with the net present values of option benefits. Finally, the major constraints to implementing a water options trading system in Australian agriculture are outlined.
2. Studies of water contracts

Options contracts are financial instruments that derive their value from another underlying asset and give the holder the right but not the obligation to buy or sell that asset. Financial options are openly traded across exchanges internationally, based on assets such as equities, indices, commodities and interest rate futures. They come primarily in the forms of calls and puts and can be American or European with respect to exercise. An American option allows the holder to exercise the option at any time until the specified date of maturity. In contrast, a European option can only be exercised on the specified date of maturity (expiry date). An American option is more expensive because of its greater probability of being exercised.

Most studies of water options contracts have been undertaken on the more mature and openly traded water markets in the United States of America, where water options offer prospects to bring positive economic outcomes. There is a substantial literature covering options contracts in the United States water market that includes Michelsen and Young (1993), Watters (1995), Villinski (2003), Ranjan, Gollehon and Aillery (2004), and Hansen, Howitt and Williams (2006). These studies have influenced the direction of research in this study to determine the applicability of options contracts to Australian water markets. Studies of options contracts in an Australian context are limited, but Page and Hafi (2007) and Hafi et al. (2005) examined water options contracts based specifically on Australian conditions. Michelsen and Young (1993) formulated an integrated hydrologic-economic model for a case study in Colorado to value a water price option. They observed that the use of multiple-exercise 'exotic' options contracts on temporary water rights is a least-cost means to provide drought insurance for urban water agencies. Six conditions were prescribed to establish water supply options contracts, based on their review of earlier literature (Young 1972; Randall 1981; Howe, Alexander and Moses 1982; Cox and Rubenstein 1985). A linear programming model was used to determine the offer price for water over a range of water supply and commodity price conditions. Although it was applied to a case study exploring the transfer of water from rural to urban users, the method is easily transferable to the scenario of evaluating the transfer of water from lower-value irrigation to high-value horticultural use and environmental flows, which is the problem addressed in this study. The major shortcoming of Michelsen and Young’s approach is their model’s failure to price the option premium. They did, however, determine that there is a positive net present value of option benefit for the transfer of water from rural to urban use for the majority of their case-study scenarios.

Watters (1995) applied the commonly used financial derivative approaches of the binomial tree and Black-Scholes (BS) methods to price call options on water in southern California. Instead of using an economic benefit model like Michelsen and Young, a stochastic dynamic programming model was applied to value multiple-exercise options contracts. He was able to conclude that call option premiums in the Rio Grande were not significantly mis-priced, suggesting that current options were written with a high probability of exercise values that in turn translate into high volatility in the option price.

Villinski (2003) also set out to value a multiple-exercise water options contract to augment water supplies during drought using dynamic programming built around a BS model. A point of interest for Australian conditions is that she explained that the BS method is an inadequate means of pricing an option in ‘nascent or thinly traded water markets’ (Villinski 2003) because a number of the basic assumptions of the BS model are violated. They include significant transaction costs, the presence of arbitrage opportunities and non-normally distributed water market prices.

Ranjan et al. (2004) addressed the issue of how to handle the dynamics between spot market transactions and options markets when conditions permit the market transfer of water. They concluded that spot and options markets serve the needs of both buyers and sellers in terms of smoothing fluctuations (reducing uncertainty) and sharing risks associated with water supply and demand. But despite the benefits that these markets offer, Ranjan et al. (2004) observed that their development has been slow to date, to which the Australian context bears strong similarities. They commented that spot markets offer higher rewards for water sellers, but these sellers also
bear larger risks due to the associated price fluctuations of these markets. This rings true of the spot market in Australia where, although sellers of water are being offered much higher rewards in the current drought, this price is subject to considerable volatility.

Hansen et al. (2006) used a simulation-optimisation approach to place a value on the transfer of water uncertainty from one party to another across several locations in California. They employed a mathematical programming framework to analyse whether increased trading among water agencies across time and space would result in significant gains from trade. The authors noted that institutional mechanisms such as water markets evolve to improve its allocation as the value of water increases. Observing the effectiveness of markets in improving the allocation of water to yield gains to trade (Howitt 1994), they suggested that more flexible institutional mechanisms such as options contracts would further improve the allocation of water. An alternative additional storage construction is thereby created to augment water supplies during times of drought. Hansen et al. (2006) used the binomial option pricing model, developed by Cox, Ross and Rubenstein (1979) and based on a distribution created in their simulation–optimisation model, to generate the option price. They suggested that there needs to be further discussion on why previous theoretical calculations of option value have exceeded option premiums on existing bilateral contracts. Their suggested reasons include mechanisms other than the market to allocate water (e.g. storage); avoiding transaction costs is another reason.

Hafi et al. (2005) considered the use of multiple-exercise European options contracts to augment wet-season flows in Australian river systems in order to create a beneficial flooding event for natural vegetation. It is related to the drought option presented above, and helps to place water options contracts in an Australian setting. Hafi et al. (2005) recognised that water transfers in Australia can be classified with respect to the permanence of trade, with permanent trade resulting in the transfer of a property right while temporary trade refers to the transfer of water allocation for any one year. They also acknowledged the disproportionate levels of risk that each of these transfer methods places on the parties involved in the transaction. Permanent transfer places a large degree of risk on the seller while temporary transfer places the risk on the buyer. Hafi et al. (2005) proposed options contracts as a potential mechanism to redistribute this risk more evenly, thereby facilitating the more efficient transfer of water. They used the pricing model presented by Michelsen and Young (1993) to value the economic benefits associated with options contracts. This model was applied to a case study on the Murrumbidgee River using a variety of exercise prices, with most scenarios presenting a net economic benefit. Their results show that water options contracts can provide economic benefit in the allocation of scarce resources under Australian conditions. The major drawback with this approach once again is its failure to establish a mechanism to price the option premium, instead assuming that the two parties to each contract negotiate the price.

Page and Hafi (2007) evaluated a drought option in Australian conditions whereby urban water supplies are augmented by options contracts on rural water. They argued that investments in excess supply capacity may be inefficient and are likely to be costly because a significant buffer of supply may be required to eliminate the effects of seasonal variability. Instead, they suggested that water options contracts may be a cost-effective means to provide a similar supply buffer for dry periods. Their model, which is also based on Michelsen and Young’s (1993) model, was used to test a number of exercise values based on the opportunity cost to irrigators. Most exercise values provide a positive net present value, meaning that the option is beneficial.

Hafi et al. (2005) and Page and Hafi (2007) show that water options contracts can provide net economic benefits by transferring water from rural use to a higher-value urban use and bringing about a more efficient allocation of water. A related question is whether there is also value in using options contracts to transfer water from lower-value broad-scale irrigation to intensive horticultural industries and other higher-value agricultural and environmental uses. This issue and the matter of assigning a premium to the option with respect to its intrinsic and time values are addressed in the remainder of the paper.
3. Introducing water options contracts in the Murray Valley water market

The European call option explored in this paper has a value based on temporary water transfer values. Hafi et al. (2005) explained that options contracts on water are allocated for the temporary use of a renewable resource while leaving the permanent entitlement unchanged. They differ from options on equities and commodities that result in the full transfer of the underlying asset. It is this transfer of temporary use of a renewable resource that creates the opportunity for an option to be exercised several times.

A second difference between water options and exchange-traded financial options is the trigger for exercise. Michelsen and Young (1993) stated that a drought-augmenting options contract on water depends on the quantity supplied, while a financial options exercise is based on the market price of the underlying asset. The interpretation is that the probability of exercise on water options contracts is based on the expected number of years with water shortage rather than being based explicitly on the asset's value with respect to the exercise price. We base the exercise of the option on water prices rather than river flows in the expectation that these prices are a good proxy for changes in river flows, but also consider the practical implications of this decision.

These differences between water options and financial options lead to the option proposed in this study being termed as exotic under the classification by Hertzler (2004). The exotic option to be tested is a package of annual European call options contracts with varying dates of maturity to give a multiple-exercise contract that is many years in length. We test packages of contracts of five years and ten years in length.

Michelsen and Young (1993) proposed six conditions to establish water options contract markets. The first of these conditions is that water supply must be reliable enough to provide sufficient water for option use during drought years. This is because the seller (writer) of the options contract is legally bound to deliver an agreed volume of water. In an Australian context, this significantly restricts the type of water licence on which an options contract can be written. Analysis of water volume allocations in the Murray River Regulated System over six years (NSW DNR 2008) shows that allocations for general-security and conveyance licences are extremely variable and therefore violate the reliability criterion. Only high-security licences exhibit reliable allocation of water and are therefore the only feasible licence class of water options for analysis.

The second criterion requires that individual property rights must be definable and transferable for market exchange (Michelsen and Young 1993). The water reform process implemented by COAG in 1995 has enabled this criterion to be satisfied.

Third, Michelsen and Young (1993) specified that agricultural operations must be capable of being temporarily suspended. The approach followed in this study for options on water involves the transfer of water from broad-scale lower-value irrigators whose activities are largely based on annual crops to horticultural operators, other producers of high-value annual crops and possibly government agencies. As long as these growers are compensated for forgone profits on the water and any fixed overhead costs in their operations, there is no reason why agricultural operations in the model could not be suspended.

The fourth condition imposed by Michelsen and Young (1993) is for both the buyer and seller of an options contract to have a realistic knowledge of water use values and alternative supply costs. Because both parties in this model are usually irrigators and water is a significant input in their business model, they should have a realistic knowledge of values and costs in order that a fair price is set. Another potential purchaser is the federal and state governments attempting to smooth river flows for environmental purposes. Staff in their agencies such as MDBC could also be expected to have a realistic knowledge of water use values and alternative supply costs.

Fifth, the probability and severity of drought (the expected probability of option exercise) must also be able to be estimated within acceptable limits of risk for both users (Michelsen and Young
The unpredictability of future weather conditions represents a significant problem in fulfilling this requirement, especially in drought-prone south-eastern Australia. Page and Hafi (2007) attempted to account for this unpredictability in calculating the net economic benefit of an option contract by eliciting their probability value from historical records. This study takes the same approach, assuming a drought probability value elicited by Page and Hafi (2007) but using 36 years of data to 2005/2006 to simulate Murray River annual flows with an exercise trigger of 80,000 ML. Of course, with climate change the use of this ‘frequentist’ approach might not be an accurate guide to future river flows.

The final criterion placed on water markets by Michelsen and Young (1993) is that the total options contract costs, including both transaction costs and transport costs to the purchaser’s point of intake, must be less than the purchaser’s next most costly water supply. In other words, in order for an options contract to be feasible, the costs of exercising the option must be less than the closest logical alternative. This is one of the two propositions tested in this study. The second proposition to test is that a financial options pricing model can be used to price the option within the bounds of this net economic benefit.

4. Methods

The methods used in the analysis are explained in detail in two appendices. The way in which we modelled the net present value of options benefit is described in Appendix 1. This step enables a decision maker to determine whether an option for water rights provides net economic benefits in the transfer of irrigation water rights between farmers who irrigate crops. The water options contract is valued from a buyer’s perspective by attempting to minimise the potential costs of augmenting water supply during periods of drought. This is a less traditional method of valuation than that used in financial markets where it is the writer of an option who determines the value of the option based on intrinsic and time values in the market.

The benefits of options trading are defined, and then we estimate the (net) present value of option benefits ($PVOB$). $PVOB$ indicates whether the option holds economic benefit. If it is positive, the options contract is a more cost-effective means of augmenting water supply during a drought. On the other hand, if $PVOB$ is negative it would be more beneficial for the buyer to pursue the alternative water source because it is cheaper than buying the options contract. This outcome would render the option worthless and the process of options trading uneconomic. The alternative source is selected to be the purchase of a high-security water entitlement of equivalent volume to that for which the options contract would be written. This is considered to be a more cost-efficient means of increasing water entitlement than building additional infrastructure.

In Appendix 2, we outline the procedure followed for modelling option prices in order to assign a definitive value to the premium that writers would demand for selling a water options contract. Two commonly used methods for pricing financial options are the binomial tree method and the formula-based option pricing models of which the most well-known is the BS method. We use the latter because the binomial tree method has a major shortcoming that is described in Appendix 2. To derive the BS model, it is necessary to make a number of assumptions that are also described in Appendix 2.

We then propose a modified BS model, termed the skewness-adjusted BS (SKABS) model, to calculate option prices to account for the positively skewed distribution of water prices. This modified model is described in section A2.4 of Appendix 2. Two water price scenarios are specified for the use of the BS and SKABS models that are based on the Murray River temporary water licence transfers over 52 months from August 2004 to November 2008. Water prices varied widely over this period, with lower prices in the first two years, punctuated by the occasional spike, and high to extremely high prices prevailing during the drought conditions during the remainder of the period. We select a range of exercise prices for options with the same expiry date that cover the range of the gross margins for the five lower-value crops used in the study (soybeans, wheat, medium-grain rice, long-grain rice and maize. The distributions of water prices are obtained using stochastic simulation (see section A2.5 in Appendix 2).
5. Results

5.1 Estimation of \( PVOB \)s

Results for the estimation of the values of options contracts for the two selected contract durations are presented in Table 1. They show that the \( PVOB \)s on irrigation water are positive for both the 5-year and 10-year contract lengths for crops with lower gross margins per ML.

This set of results shows two things. First, the \( PVOB \)s of options contracts for the transfer of water diminish as the exercise price increases, as expected. Second, options contracts facilitating transfer from low-value water uses to higher-value uses provide positive \( PVOB \)s while those from a high-value use result in net economic costs. Another significant result from this analysis is the increased \( PVOB \) of a longer option period. A 10-year option provides greater economic benefit than one with a 5-year duration, which is logical because it provides the buyer with greater flexibility. But a 5-year option provides greater \( PVOB \)s on a yearly basis.

Table 1 Present Values of Options Contracts for Different Contract Durations

<table>
<thead>
<tr>
<th>Crop</th>
<th>Exercise price ($/ML)</th>
<th>( PVOB ) 5-year contract ($/ML)</th>
<th>( PVOB ) 10-year contract ($/ML)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soybeans</td>
<td>62.89</td>
<td>289.51</td>
<td>496.57</td>
</tr>
<tr>
<td>Wheat</td>
<td>159.44</td>
<td>124.08</td>
<td>212.70</td>
</tr>
<tr>
<td>Medium-grain rice</td>
<td>110.91</td>
<td>206.57</td>
<td>354.29</td>
</tr>
<tr>
<td>Long-grain rice</td>
<td>128.97</td>
<td>186.71</td>
<td>316.98</td>
</tr>
<tr>
<td>Maize</td>
<td>188.54</td>
<td>73.96</td>
<td>126.73</td>
</tr>
<tr>
<td>Broccoli</td>
<td>463.43</td>
<td>-410.17</td>
<td>-1798.69</td>
</tr>
<tr>
<td>Potatoes</td>
<td>1279.03</td>
<td>-1309.70</td>
<td>-3086.00</td>
</tr>
</tbody>
</table>

5.2 Estimation of water option prices

In this section, an attempt is made to provide a fair value for an option premium by applying the BS and SKABS models to water temporary transfer values at the beginning of the option period. The BS model was initially used to calculate water option prices for the two scenarios in order to provide a comparison with the results obtained using the preferred SKABS model. The two higher-value crops, broccoli and potatoes for processing, are excluded from the list of crops because they have negative estimated \( PVOB \)s.

5.2.1 Using the BS model to calculate option prices

Results for the 5-year and 10-year options show that the relatively high volatility and unusually high prices for water transfer in recent years cause the estimated option prices generally to exceed \( PVOB \)s in both scenarios. The estimated option prices increase at a decreasing rate as the level of volatility is raised. Prices for the 10-year options are considerably higher than those for the 5-year options.

5.2.2 Results of stochastic simulations using the SKABS model

The SKABS model was successfully estimated, with mean option prices clearly higher than those obtained using the BS model. These prices increase at an increasing rate as the level of volatility is raised, in contrast to the BS results. Table 2 contains the mean sample option prices and those option prices as a proportion of \( PVOB \)s for the stochastic simulations of the 5-year and 10-year option contracts for the two volatility indices. In all four scenarios, the mean sample option prices
vary narrowly across the different exercise prices. Option prices are therefore not very sensitive to the choice of exercise price at these high levels of volatility. Nor are they particularly sensitive to the choice of discount rate. But the scenarios do vary markedly between each other: option prices increase substantially with higher volatility and longer contracts.

Table 2 Results of Stochastic Simulations Based on Data for Scenarios 1 and 2 for 5-Year and 10-Year Options: SKABS Model

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Mean sample option price ($/ML)</th>
<th>Option price as a proportion of PVOB (%)</th>
<th>Mean sample option price ($/ML)</th>
<th>Option price as a proportion of PVOB (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-year options:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soybeans</td>
<td>671.25</td>
<td>598.5</td>
<td>2890.33</td>
<td>1068.4</td>
</tr>
<tr>
<td>Wheat</td>
<td>680.19</td>
<td>222.5</td>
<td>2893.02</td>
<td>586.1</td>
</tr>
<tr>
<td>Medium-grain rice</td>
<td>682.27</td>
<td>278.6</td>
<td>2893.62</td>
<td>537.9</td>
</tr>
<tr>
<td>Long-grain rice</td>
<td>692.93</td>
<td>307.1</td>
<td>2896.53</td>
<td>398.9</td>
</tr>
<tr>
<td>Maize</td>
<td>684.53</td>
<td>331.0</td>
<td>2894.26</td>
<td>496.3</td>
</tr>
<tr>
<td>10-year options:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soybeans</td>
<td>1890.15</td>
<td>2577.0</td>
<td>772.62</td>
<td>5893.9</td>
</tr>
<tr>
<td>Wheat</td>
<td>1893.60</td>
<td>946.2</td>
<td>764.34</td>
<td>3227.7</td>
</tr>
<tr>
<td>Medium-grain rice</td>
<td>1894.38</td>
<td>1181.6</td>
<td>768.21</td>
<td>2960.8</td>
</tr>
<tr>
<td>Long-grain rice</td>
<td>1898.26</td>
<td>1283.6</td>
<td>766.71</td>
<td>2191.5</td>
</tr>
<tr>
<td>Maize</td>
<td>1895.22</td>
<td>1399.5</td>
<td>762.21</td>
<td>2730.6</td>
</tr>
</tbody>
</table>

These scenario estimates reflect how strongly the level of volatility would influence water option prices were options trading to be introduced in the present circumstances. All sample means of option prices are multiples of the PVOBs, making the purchase of options an unattractive proposition. They reflect the relatively high levels of ‘in the money’ events associated with the high level of price volatility and high spot prices during the study period.

1 Similar effects can be viewed on Hertzler’s (2008) option pricing calculator for an increase in the volatility index (σ) in the differential equation, \( w_t + w_x(\alpha x + \beta) + 0.5w_{xx} \sigma^2x^2 - rw = 0 \). The default parameter of \( \alpha = -1 \) is used and it is initially assumed that \( r = 0.4, \sigma = 0.5 \) and both the spot and exercise prices are scaled to 1.0 in a normal year for an ‘at-the money’ single-year call option on a commodity (Hertzler 2003, p. 17). Given these parameters for a single-year option, the option price is about 0.15 at the beginning of the period (\( t = 0 \)), similar to the approach used for the SKABS model. An increase in \( \sigma \) to 1.6 causes the option price to increase to 0.55, more than half the scaled spot price of the commodity.
Figure 1 shows the surplus of PVOBs over the estimated option prices in the 5-year model for a volatility range from 0.5 to 1.6. Once a volatility index of 0.7 is reached, all sample mean option prices exceed the respective PVOBs. They then rapidly become greater than the PVOBs. As a volatility index 0.5 did not occur at any stage during the study period, and is unlikely occur for lengthy periods in the future, it is expected that option prices would exceed PVOBs most of the time. At a volatility index of 1.0, the discrepancy between PVOBs and premiums is around $500/ML for all crops.

![Figure 1 Surplus of PVOBs over option prices in the 5-year SKABS model.](image)

6. Discussion

6.1 The obstacle of high premiums

A continued high level of price volatility in the Murray River water market is likely to render option prices well in excess of PVOBs. It is concluded, therefore, that the major factor behind the lack of opportunities for irrigators to benefit from buying a call option contract in water is likely be the persistently high levels of price volatility. Were price volatility not to be so great, prospects for creating a market for water options would improve greatly. The results using the SKABS method in water option markets not subject to recent levels of drought stresses are difficult to judge given the abnormal climatic conditions prevailing for most of the study period. Perhaps it would yield a pricing result within the economic benefit generated by the option and not far from the PVOBs if climatic conditions were stable with plentiful rainfall. It is of interest, though, that the price volatility index was still high during the first two years of the study period when seasonal conditions were relatively favourable and spot water prices were low.

As mentioned above, the manner in which exercise prices have been set is prone to error. But the option price is insensitive to the exercise price except at volatility levels well below those prevailing over the study period. The range of exercise prices was set from $200/ML to
$1200/ML, which is slightly greater than the range of marginal values of high-security water used by Page and Hafi (2007) for setting exercise prices in their study. When the volatility index is set at 1.0 and 1.6, the response in option price to a change in exercise price is negligible over the full range of spot water prices experienced during the study period. It is only at volatility levels of 0.5 and below that a discernible increase in option price occurs in response to a fall in the exercise price.

6.2 Can option premiums be reduced?

High water price volatility makes writing an options contract unattractive unless the writers receive high premiums, but high premiums will discourage buyers of options from entering the market. The use of barrier options is one way to reduce option premiums when high levels of water price volatility prevail. The payoff of a barrier option depends on whether the price of the underlying asset reaches a certain level during a designated period. Hull (2006) describes these options as knock-in and knock-out options whereby they either come into existence or cease to exist when the underlying asset price reaches a certain barrier. This method could be useful in reducing the risk associated with the large positive tail of water price distributions and thereby reduce the premium on the option demanded by the writer. A second benefit of a barrier option would be if the barrier were to apply to water allocation. The option would terminate if, say, allocations on licences were to drop below 80 per cent. This would reduce the risk of the option writer not being able to provide the contracted water, and thus reduce the risk premium they must be paid to sell such an option.

A second way to reduce option prices is to limit the number of available option exercises to less than the total number of potential exercise dates. An example of this approach would be a 10-year multiple exercise call option that allows a maximum of five exercises over the lifetime of the contract. The benefit of such a contract would be the cap placed on the probability of exercise that would result in a reduction in the risk premium demanded by the writer. Villinski (2004) proposed such a contract, with the option to be valued through the use of stochastic dynamic programming based on the BS formula.

6.3 Other obstacles to implementing water options contracts

Even if options contracts on water provide positive economic benefits through the reallocation of scarce water resources in Australian water markets and premiums are kept within a range attractive to buyers, a number of obstacles need to be overcome for successful options trading. Stochastic simulation enables us to estimate option prices by applying a SKABS model without concern for the violation of a number of conditions that would nevertheless affect the actual implementation of options trading in an Australian context. Chief among these conditions is continuous trading in the spot market, the presence of random walk in the spot market, an absence of riskless arbitrage opportunities in the spot market, the absence of transaction costs, standardised option contracts and the presence of a suitable trigger mechanism for option exercise.

The thinness of current water markets is perhaps the most difficult obstacle to overcome. With thin trade, arbitrage opportunities arise for investors to buy at one price and sell immediately for a higher one in the water market.

The assumption of a random walk in spot water prices is dubious. The process of formation of water prices is significantly more complex than implied by this assumption because these prices fluctuate seasonally and in response to weather forecasts. We undertook a series of tests of the random walk assumption in the spot water price series for each year of the study period and it was strongly rejected, casting doubt on the efficacy of both the BS and SKABS methods to serve as a practical means to determine option prices.

The assumption that transaction costs are non-existent is unrealistic. Colby (1990) observed that water exchanges worldwide are subject to regulation or constraint in some form, and this intervention imposes costs in the marketplace caused by greater uncertainty for both buyers and
sellers. In the Australian context, Martin et al. (2008) concluded from their study of water markets that they do not work effectively, resulting in high transaction costs. First, numerous regulations, market instruments and organisations make water transactions more complex than they should be. Second, the complexity of these arrangements is exacerbated by political and administrative interests in water as property. Third, there are impediments to obtaining mandatory licences and governments frequently make alterations to planning and administrative arrangements. Finally, conflict is generated from competition between institutions, and effective coordinating mechanisms are absent. Martin et al. (2008) offer some useful suggestions to improve the operation of the water market that in turn would make it easier to introduce trading in water options contracts.

Options contracts would need to be standardised and an acceptable means available to calculate a fair option price to keep transaction costs within reasonable bounds. Otherwise, each contract would need to be tendered out in order to value the premium, resulting in significant transaction costs in market operations due to the large burden of collecting information for this process. Page and Hafi (2005) recommended the standardisation of terms and conditions for all options contracts. Each water call option would need to state: the exercise price of the option; date of maturity; total volume of water for which the contract is established; potential strike dates within the life of the option (set to the same date each year); option premium; and a clause to cover any government intervention affecting the water entitlement on which the option is based. By specifying each of the above, the burden of information gathering would be decreased thereby reducing the costs associated with each transaction. Page and Hafi (2005) suggested that an options market parallel to the current spot market could be put in place for the trade of options that is overseen by an independent government body or existing exchange such as the Sydney Futures Exchange.

Michelsen and Young (1993) mentioned other ways to improve the terms and conditions of an options contract. They suggested the insertion of a clause for the right of first refusal. Should the option writer decide to sell the water entitlement before the termination date of the contract, the holder of the option would have the right to match the offered price. Their second suggestion was the inclusion of an escalation clause in the agreement to protect sellers against the effects of inflation.

While water itself might be a reasonably homogeneous commodity (quality variations notwithstanding), spatial and temporal variations in its availability also make pricing water options contracts problematic. In particular, setting up a trading market could encounter significant practical difficulties in the absence of an acceptable trigger mechanism for exercising an option. The trigger mechanism needs to include an appropriate criterion for water shortage and where in the Murray Valley this shortage is occurring. It should also be able to cater for the seasonality in water supply and water demand for different crops, especially the different seasonal requirements of winter and summer crops and the limitation on a European option buyer to exercise it only at one specified period in the year.

A critical issue to tackle is whether the trigger mechanism should be price-based or volume-based. Pricing methods currently use a price-based trigger mechanism, in line with the approach adopted for options in financial securities that is followed in this study. A volume-based mechanism is preferable for water options but is more difficult to formulate and to operate.

Finally, Ranjan et al. (2004) described options markets as a means to hedge against water price fluctuations, but observed that they still place substantial risks on farmers participating in these markets. Translated to an Australian context, this risk will present substantial resistance to participation in water markets if large fluctuations persist in water supplies, thus slowing participation in both spot and option markets.

Effective pricing is necessary for efficient trading in options. It is conceivable that future modelling advances will require fewer restrictions on their application in a water market than the existing models described above, including the SKABS model, by overcoming the above violations. Two
examples of advanced modelling techniques that may be forerunners of better models in the future are presented in Appendix 2. The model with the greatest prospect of achieving this aim is a stochastic dynamic programming model that is structured to handle multiple-year options contracts.

6.4 Main challenges

Three main challenges face agribusinesses interested in reducing the risk of drought in agricultural production by developing and operating derivative markets for water. The first challenge is to reduce the level of premiums that are likely, on current estimation, to be too large to induce farmers to enter such markets in sufficient numbers to make them workable. While rainfall patterns are only likely to become more uncertain with climate change and exacerbate the problem, the solution is to reduce fluctuations in river flow through infrastructure investments. However, it is likely to be an expensive one and may conflict with the need to maintain environmental flows. The second challenge is to keep transaction costs as low as possible in trading water options contracts. Third, the peculiar nature of water derivatives requires the use of sophisticated modelling, which demands research into possible methodological advances in pricing water options. Each of these three challenges will be testing. But without all three of them being successfully met, the prospects for developing a trading system in water options in Australia appears to be bleak.

7. Conclusions

As water resources become scarcer in Australia, institutional mechanisms involved in their allocation will have to adapt. The process to separate land and water property rights that was begun in 1995 created the opportunity for water entitlements to be traded on the open market. In the years following this policy change, the water market increased in popularity and now forms the primary means of price discovery for water transfers.

Although an open market has increased the allocative efficiency of water over the past decade, there is still room for instruments such as options contracts to increase the efficiency of resource allocation through the redistribution of the risks borne by buyers and sellers in the open market. The beneficial influence on water resource allocation in the more highly developed water markets of the United States of America suggests it is possible to introduce options contracts to facilitate trade in local water markets. In this study, similar results have been reported with water options contracts creating positive \( PVOBs \) in the trade of water from lower-value broad-scale irrigation to greater-intensity high-return horticultural pursuits and environmental flows.

The BS and SKABS models were employed to value water option prices for comparison with \( PVOBs \). The SKABS model was preferred because of the need to take account of non-normality in the distribution of high-security water returns in the Murray River market that was chosen for a case study. Given the divergence in values between the BS and SKABS models, use of the BS formula is likely to undervalue option prices considerably at all but unrealistically low levels of volatility in water prices and therefore would not be a pricing model that provides a definitive ‘fair value’ on the option premium. Both models were found to price options within the range of \( PVOBs \) at low price volatility, but they were unsuccessful in doing so when price volatility is in the range experienced over the study period.

In addition to the very high option premiums with high water price volatility, other obstacles are likely to impede progress in implementing a water options trading scheme in Australia. Chief among these obstacles are the thinness of the water market, an absence of random walk in water returns, difficulties in standardising water options contracts and problems in designating an appropriate trigger mechanism for exercising option contracts. In order for options trading to be effectively implemented in an Australian context, a model needs to be developed that accurately assigns fair intrinsic and time values of water options contracts and is accepted by buyers and sellers.
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Appendix 1: Modelling the net present value of options benefit

The aim of the first step in the analysis is to determine whether an option for water rights provides net economic benefits in the transfer of irrigation water rights between farmers who irrigate crops. The approach put forward by Michelsen and Young (1993) is followed, with one main difference. Whereas Michelsen and Young used a linear programming model to estimate offer prices for water over a wide range of conditions, we use estimates of gross margin per megalitre (ML) of water for individual irrigated crops grown in the Murray Valley. Our reason for doing so is that the farm conditions vary widely in the region and a vast number of representative farms would need to be modelled to provide relevant linear programming solutions to obtain a range of offer prices.

The use of available gross margins for a range of crops grown in the region is likely to provide estimates of farm profitability per ML of water used that are almost as relevant as offer prices derived from linear programming solutions. A potential shortcoming in using gross margins is that they are average returns to the use of irrigation water rather than the preferred measure of marginal returns.

The water options contract is valued from a buyer's perspective by attempting to minimise the potential costs of augmenting water supply during periods of drought. The model proposed by Michelsen and Young (1993) is to value an options contract as a function of the cost of an alternative water supply minus the expected exercise cost and appreciation of the alternative supply. This approach is denoted in the following formula:

\[ P_{VOB} = \sum_{t=0}^{T} [(K_{t=0} + r + M_t) - (X^*P)]d_t + [K_{t=0} - K_{t=0}(1+a)]d_t \]  

(1)

where \( P_{VOB} \) is the (net) present value of option benefits, \( t \) is a particular year of the contract, \( T \) is the contract termination year, \( K_{t=0} \) is the capital cost of alternative supply at the beginning of the option term, \( r \) is the risk-free real rate of interest, \( M \) is the annual maintenance cost of the alternative, \( X \) is the exercise (strike) price of the option, \( P \) is the probability of option exercise in a year \( (0 \leq P \leq 1) \), \( d_t \) is the discount factor to calculate present value, \( 1/(1 + r)^t \), and \( a \) is the annual rate of appreciation (or depreciation) of the alternative supply of water.

The cost of an alternative water supply can in turn be decomposed into two factors. The first factor is denoted by the capital cost of owning the alternative for the option period \( (K_{t=0} + r) \), where \( K \) is the capital outlay to gain access to a megalitre (ML) of water and \( r \) is the discount rate that accounts for the opportunity cost of capital investment in this source of water. Effectively, this can be thought of as the interest cost of owning the asset given the entitlement is bought in the first year and sold for the same value at the end of hypothetical option period. The capital outlay on an additional high-security allocation in the Murray River Regulated System has been valued at \$1423/ML. This figure was obtained by taking the weighted-average permanent transfer price per ML over three years (NSW DNR 2008). The real risk-free discount rate assumed in this model is set at 4 per cent per annum, based on data on government bond yields in Australia over the past two decades. The second cost of alternative supply is its annual maintenance cost, \( M \). Owning another licence has certain fixed costs associated with it that are assumed to rest on the owner of the licence. These fixed costs would include items such as government levies, asset maintenance and renewal funds. Their value, estimated by the NSW DPI (2007) at \$14.02/ML, represents the total fixed water costs per hectare assuming 100 per cent allocation in the Murray River Regulated System.

The method to allocate a value for the option exercise price has been addressed in the literature. Michelsen and Young (2003) set the exercise price at the level that farmers would be willing to release water supplies or, in other words, a value that would fully compensate them for forgoing the benefits of using that water. Page and Hafi (2007) adopted this procedure to value the exercise price at the marginal value of high-security water at drought allocations, using a range of exercise prices from \$200/ML to \$1100/ML. This interpretation follows Just, Hueth and Schmitz.
(1982), who identified the extent to which writers of options should be compensated for forgone incomes as a result of sacrificing their water entitlement. It is assumed that the writer must be reimbursed for both the profit that would have been earned on the water and any capital and fixed costs to be met whilst forfeiting access to that water. Gross margins per ML are used as a profit measure for rice, wheat, maize, soybeans, broccoli and potatoes (for processing) in the model to provide a range of exercise prices. The gross margins are estimated by stochastic simulation in Simetar (Richardson, Schumann and Feldman (2006, p. 14) assuming triangular distributions for crop yields and prices, based on data published by NSW DPI (2007). For consistency, data on estimated crop yields and prices are for the period of the study rather than current estimates. Among variable costs, irrigation water costs were treated as stochastic to reflect the opportunity cost of water in the temporary water transfer market while other variable costs were assumed to be deterministic. Ten thousand samples were run and mean sample gross margins were calculated to give the exercise values presented in Table A1.

Table A1 Assumed Exercise Values for the Water Option Contracts

<table>
<thead>
<tr>
<th>Crop</th>
<th>GM/ML forgone ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-grain rice</td>
<td>128.97</td>
</tr>
<tr>
<td>Medium-grain rice</td>
<td>110.91</td>
</tr>
<tr>
<td>Maize</td>
<td>188.54</td>
</tr>
<tr>
<td>Soybeans</td>
<td>62.89</td>
</tr>
<tr>
<td>Wheat</td>
<td>159.44</td>
</tr>
<tr>
<td>Broccoli</td>
<td>463.43</td>
</tr>
<tr>
<td>Potatoes for processing</td>
<td>1279.03</td>
</tr>
</tbody>
</table>

Source: Adapted and updated data from NSW DPI (2007).

The most applicable study to obtain a value for the probability that the option will be exercised in any year in Australian conditions is by Page and Hafi (2007). Assessing the feasibility of options contracts for rural to urban transfer, they estimated the probability of exercise in a year as the expected number of three shortages for Canberra over a ten-year period. The same P value of 0.3 is used as the basis for calculating the potential economic benefit of water options contracts in this study.

The final component of the model is the forgone capital appreciation of the alternative source of water, $K_{t_0} - K_{t_0}(1+a)^T$. The expected sale price of the licence in five or ten years should be subtracted from the initial purchase price to calculate the missed capital returns. The licence is expected to appreciate in value during the ownership period, as illustrated by Hafi et al. (2005) for the Murrumbidgee River Regulated System. Over the previous 14 years, general-security water licences in the system increased in value by an average of 5 per cent per year. Research on the Murray River Regulated System indicates that high-security licences increased in value by 4.5 per cent per year over the three years to 2007 (NSW DNR 2008). Countering this appreciation would be a commensurate increase in the opportunity cost of temporary water licences, which should be taken into account when calculating crop gross margins. Michelsen and Young (1993) asserted that water rights can be assumed to be non-depreciating assets, but structural alternatives with limited lifetimes must be depreciated and permanent water licences rely on such structures. In sum, there is no easy way to estimate changes in capital value in our case study area and so the approach followed in this study is to assume no capital appreciation or depreciation.
Appendix 2: Options pricing

A2.1 Modelling option prices

A problem with the model presented by Michelsen and Young (1993) is its failure to assign a definitive value to the premium that writers would demand for selling a water options contract. Michelsen and Young (1993) suggest that the purchaser (who values the option by the PVOB) and writer must come to some agreement on this value. This approach could lead to market failure caused by asymmetry in the information available to each user, and difficulties in bringing suitable parties together and keeping transaction costs low. The second step in this paper aims to overcome this deficiency by applying a financial-based options model to assign a value to the intrinsic and time value of the option. If this method is successful, the option premium derived from the pricing formula will provide a comparison with the PVOB that was calculated when estimating the values of options contracts.

Two commonly used methods for pricing financial options are the binomial tree method and the formula-based option pricing models of which the most well-known is the BS method. According to Hull (2006), the binomial tree is a useful and popular means to price an option by constructing a decision tree mapping the different potential paths that the price of the optioned asset may take. This method was developed by Cox, Ross and Rubenstein (1979) and presents a simple discrete-time pricing formula valuing the option through arbitraging. Benninga and Wiener (1997) and Hansen et al. (2006) described the binomial option price as converging towards that of the BS price as the number of iterations in a simulation increases. Since the number of iterations in the binomial tree would increase its accuracy, the BS model could be substituted as the pricing model. The binomial tree method has its shortcomings, notably difficulty in reaching convergence even when the model is conditionally stable, which is why the BS model is usually preferred (Greg Hertzler, personal communication, 18 October 2013).

One potential problem for options price formation is the absence of a ‘hidden’ martingale no-arbitrage restriction on the expected future asset price, of the type recently outlined by Corrado (2007). Corrado (2007, p. 528) introduced a hidden restriction ‘via a reduction in parameter space for Gram-Charlier expansions calibrating the expansion’. But his application of the restriction in an example appears to have had minimal effects on fitted call option prices.

A2.2 The BS model

The BS model, developed by Black and Scholes (1973) and extended by Merton (1973), follows arguments similar to the no-arbitrage assumption used in the binomial tree approach whereby a riskless portfolio is set up, consisting of positions in both the derivative and asset markets (Hull 2006). In the absence of arbitrage opportunities, the return from the portfolio must be the risk-free rate of interest, thus leading to the Black-Scholes-Merton differential equation (Hull 2006).

Hull (2006) should be consulted for a comprehensive overview of the derivation of the Black-Scholes-Merton differential equation. The formula derived by Black and Scholes (1973) and Merton (1973) to price a European call option at time 0 on a non-dividend-paying stock (also applicable to other underlying assets) is:

\[ c = S_0N(d_1) - Xe^{-rT}N(d_2) \] (2)

and

\[ d_1 = \frac{\ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \] (3)

\[ d_2 = d_1 - \sigma\sqrt{T} \] (4)

where \( c \) is the European call price (or premium), \( S_0 \) is the stock (asset) value at time zero (beginning of the option period), \( \sigma \) is a price volatility index, \( T \) is the time to maturity of the option in years, and \( N(.) \) is the cumulative normal distribution function.
The European call price, represented by $c$, is the option premium that the writer would charge the purchaser in compensation for forgoing their entitlement to use the irrigation water in a given period. This concept is in theory extendable to the price of any asset; in this study, it is applied to water at the beginning of the option period.

The volatility of the given stock (or other asset) price denoted by $\sigma$ can either be estimated from historical price data or implied from the pricing of similar options. We are examining a proposal for a real option that currently does not exist in the Australian water market, and it is difficult to find other options of the same nature to determine the implied volatility. We therefore have to rely on the estimation of volatility using historical data on water prices. SITMO (2007) gives the most commonly used formula to calculate volatility from historical data that is followed in this study, with a slight modification in the annotation. The so-called ‘close-to-close’ formula is amended to a weekly ‘average-to-average’ formula because a data series on a daily basis is unavailable and transactions are not always recorded on the last day of the week for the weekly data (data are often only available for one day during the week). The formula is defined by:

$$\sigma = \sqrt{\frac{Z}{n-2} \sum_{i=1}^{n-1} (r_i - \bar{r})^2}$$  \hspace{1cm} (5)$$

$$r_i = \ln\left(\frac{C_{i+1}}{C_i}\right)$$  \hspace{1cm} (6)$$

$$\bar{r} = \frac{r_1 + r_2 + \ldots + r_{n-1}}{n-1}$$  \hspace{1cm} (7)$$

where $C$ is the average water price in the $i$-th week, $n$ is the number of weeks of observations used in the volatility estimate, $r_i$ is the return in week $i$, and $Z$ is the number of weeks in a year (52 given the use of weekly data). Because there is negligible trade in water in July and no estimate is made of $r_i$ between years beginning in August, setting $Z$ at 52 weeks is equivalent to assuming there is no volatility during the non-trading weeks. This approach is similar to that used for estimating volatility per annum in equity markets where volatility is assumed to be zero during non-trading days (Hull 2006). Equation (6) shows that water returns are calculated as the change in log price from one week’s average water price to the next week’s average price.

For the models estimated in this study, 5-year and 10-year option terms are used for $T$. The 5-year time period means that five separate European calls are packaged to calculate a value for this multiple-exercise option. The first call has one year to maturity, the second two years to maturity, and so on to the fifth having five years to maturity. In the case of a 10-year option, there would be 10 separate European calls. Therefore, each contract has a separate premium attached and the total value of the option is the sum of all premiums discounted to present value.

Finally, the cumulative normal distribution function is applied to $d_1$ and $d_2$ (Hull 2006). It is, in effect, the probability of the option expiring ‘in the money’ (that is, with the water spot price greater than the exercise price) and therefore being exercised, given that the underlying water returns are normally distributed.

### A2.3 Assumptions of the BS model

In deriving their model, Black and Scholes (1973) and Merton (1973) drew on seven assumptions of varying degrees of strength. First, the asset returns (returns to water prices in this study) are normally distributed as noted above, implying that water prices have a log-normal distribution (Hull 2006). Clearly, the temporary transfer prices of irrigation water over the 52 months of the data, on which the model is based, violate this assumption. Figure A1 illustrates that the
distribution is strongly positively skewed, representing the impact on water values of current
drought conditions.


**Figure A1** Distribution of average weekly prices of Murray River temporary water licence transfers, August 2004 to November 2008.

Although it represents the water market situation over less than five years, such a highly skewed
distribution is likely to reflect the long-term structure of the market, with regular demand for water
in most years interspersed with occasional spikes in prices caused by dry conditions. It is partly
leptokurtic, in that the right-hand side of the distribution has a thick tail, and partly platykurtic, in
that the left-hand side of the distribution has a very short tail. The assumption of a log-normal
distribution is clearly violated. The distribution also appears to be trimodal although the relatively
short study period may be responsible for this structure. This market pricing structure results in
returns to water that are also positively skewed (at 0.114, nowhere near as strong as for water
prices at levels) and have a high kurtosis value (at 4.188, substantially higher than the value of 3
for a normal distribution). The positive skewness is different from the expected negative
skewness that is typically associated with stock returns in equity markets (e.g. Heston and Nandi
Engle and Mancini 2008).

The second assumption is that prices follow a geometric Brownian motion with constant volatility
and drift, whereby changes in an asset’s price follow a continuous-time, non-stationary, Markov
process. To satisfy the conditions of the Markov property, only current prices can be relevant in
predicting future prices. Brownian motion is related to a random walk in the sense that, after
many steps, a random walk converges to a Brownian motion. This assumption is also dubious,
and is discussed in detail below.

The third key assumption is that no riskless arbitrage opportunities arise and security trading is
continuous. The best way to satisfy this assumption is for the market to have a high volume of
transactions on a regular basis, a feature that was conspicuously absent in the Australian water
market in the study period. Other assumptions are that transaction costs and taxes are non-
existent; the asset should be able to be short sold with the full use of the proceeds permitted; all
securities are perfectly divisible; there are no dividends during the life of the derivative; and the risk-free rate of interest, $r$, is constant and the same for all maturities.

The egregious violation of the assumption of normal distributions of $d_1$ and $d_2$ in the BS formula (equation (2)) is of particular concern. Due to its violation, the BS method is an unsuitable model for pricing irrigation water options in the Murray River water market. Adoption of the alternative skewness-and-kurtosis-amended BS (SKABS) model is a way to accommodate these non-normal distributions. We return below to the violation of other assumptions, notably the obstacles presented by the presence of transaction costs and arbitrage opportunities, and concern about the validity of the assumption of Brownian motion.

A2.4 Modifying the Black-Scholes model for pricing water

Various attempts have been made to modify the BS model to calculate option prices to account for non-normality by including terms to account for the positively skewed distribution of water prices and its leptokurtic properties. Examples of the density-expansion approach cited by Corrado (2007) include, among others, Jarrow and Rudd (1982), Corrado and Su (1996), Heston (1993), Ane (1999), Jondeau and Rockinger (2000), and Chauveau and Gatfaoui (2002). The skewness-and-kurtosis-amended BS model, termed the SKABS model by Vähämäa (2003, p. 10), was developed by Corrado and Su (1996) and is well suited to this study. It is based on a Gram-Charlier expansion of the standard normal density function to adjust for skewness and kurtosis, and was amended by Brown and Robinson (2002) for an errant minus sign. This amended version is used to apply the SKABS model using the closed-form pricing formula (Vähämäa 2003, p. 7):

$$c = S_0 N(d) - X e^{-rT} N(d - \sigma \sqrt{T}) + \mu_3 Q_3 + (\mu_4 - 3) Q_4$$

(8)

where:

$$Q_3 = \frac{1}{3!} S_0 \sigma \sqrt{T} [2 \sigma \sqrt{T} - d] n(d) + \sigma^2 T N(d)$$

(9)

$$Q_4 = \frac{1}{4!} S_0 \sigma \sqrt{T} [(d^2 - 1 - 3 \sigma \sqrt{T}(d - \sigma \sqrt{T})) n(d) + \sigma^3 T^{3/2} N(d)]$$

(10)

$$d = (\ln(S_0/X) + (r + \sigma^2/2) T)/\sigma \sqrt{T}$$

(11)

The standard normal density function is designated by $n(.)$, and $\mu_3 Q_3$ and $(\mu_4 - 3) Q_4$ measure the effects of skewness and kurtosis, respectively, on the option price. Other notation is as previously defined. The calculation of $d$ is equivalent to the calculation of $d_1$ and $d_2$ in the Black-Scholes formula. Vähämäa (2003, p. 7) explained that this formula ‘is particularly convenient from a hedging point of view since it yields closed form solutions for the hedge ratios’.

A2.5 Stochastic simulation using the BS and SKABS models

Two water price scenarios are specified for the use of the BS and SKABS models. In the first scenario, a price volatility index of 1.0, or 100 per cent, is used, being the lowest estimate of the index over the study period. In the second scenario, a volatility index of 160 per cent is used that represents the historical data over the whole study period. These high indices contrast with all examples used by Hull (2006) for which the volatility figure was less than 40 per cent.

The same distribution of water prices was assumed for each index, based on the average weekly prices over the period. The kernel density-estimated random variable in Simetar was employed to obtain 10 000 samples of spot water prices. This simulation procedure adopts ‘Parzen type kernel density estimators [Gaussian type] to evaluate a smoothed value that represents a point on the cumulative distribution function (CDF)’ (Richardson, Schumann and Feldman 2006, p. 19). The kernel density-estimated random variable was chosen among other sample-based distributions and the standard log-normal distribution that is assumed for underlying asset prices in the
standard BS model. Selection was based on a comparison of CDFs using a scalar measure to compare the difference between the actual and fitted CDFs by calculating ‘the sum of squared differences between two CDFs with an added penalty for differences in the tails’ (Richardson et al. 2006, p. 37). Stochastic independence is assumed between the enterprise gross margins used for exercise prices and the spot price of water in both scenarios. A range of exercise prices is commonly used for options with the same expiry date, and we assume that they cover the range of the gross margins for the five lower-value crops used in this part of the study.

We undertook a series of Ljung-Box tests of autocorrelated errors in the spot water price series for each year of the study period. For each annual series, the test for white noise was strongly rejected. It appears that a combination of autoregressive and moving average structure exists, with the nature of this structure varying between years, violating the random walk assumption. If this assumption is badly violated, it would cast doubt on the efficacy of both the BS and SKABS methods.
Appendix 3: More advanced modelling methods

A3.1 Stochastic volatility models

A shortcoming of both the BS and SKABS models for practical use in pricing options is their underlying assumption of constant volatilities over time. Stochastic volatility models have been used to overcome this deficiency and to accommodate asymmetry in the distribution of asset returns. Early models in this genre were exponentially weighted moving average (EWMA) models and autoregressive conditional heteroskedasticity (ARCH) models (Hull 2006). Generalised autoregressive conditional heteroskedasticity (GARCH) models are the most advanced of these models for pricing options that overcome the problem of constant volatility by tracking variations in volatility through time. GARCH (1,1) models are preferable to EWMA models because they incorporate mean-reversion (randomly varying volatility rates pulled back to a long-term mean level) (Hull 2006). Whereas the SKABS model deals with non-normal distributions in an ad hoc manner by assuming a given level of volatility, GARCH models handle them in an internally consistent way.

Barone-Adesi et al. (2008) provide a good summary of the different sorts of GARCH models that have been developed over the past two decades, culminating in their own model that features filtered historical simulation that is arguably the most sophisticated GARCH model currently available. A major early landmark in the literature on GARCH models is Heston and Nandi (2000) who noted that the GARCH model approximately converges to the so-called ad hoc BS model proposed by Dumas, Fleming and Whaley (1998) under certain strong assumptions. According to Heston and Nandi (2000, p. 433), the GARCH model improves on the BS model because of its ability ‘to simultaneously capture the correlation of volatility, with spot returns and the path dependence in volatility’. Heston and Nandi’s (2000) model has disadvantages. It assumes ‘normal return innovations, a linear risk premium, and the same GARCH parameters for historical and pricing asset returns’, according to Barone-Adesi et al. (2008, p. 1225) who ran Monte Carlo simulations enabling them to relax these assumptions. Their GARCH option pricing model with filtered historical simulation is the preferred approach, because it allows for ‘different distributions of historical and pricing return dynamics’ (Barone-Adesi et al. 2008, p. 1224).

The GARCH models have typically been estimated for data series of variables in financial markets that are very deep, a feature markedly lacking in the case of Australian water markets. Unfortunately, the limitations of the spot water price data, brought about by market thinness, mean that there are frequently days of no trade. Also, time-series analysis indicates that different autocorrelated error structures existed in the spot water market in each year of the study period, making the application of such a model highly problematic with small annual data sets given a break of a month between irrigation years. Because their practical allocation in Australian water markets is stymied by the nature of the spot water price data and different autocorrelated error structures in each year requiring re-estimation at the beginning of each year, the BS and SKABS models are estimated despite their shortcomings in respect of the measure of volatility and violations of other assumptions. Of these two models, the latter is the preferred approach. We ran these models at different levels of volatility to cover the range of estimates during the study period by simulating spot water prices for 10 000 samples based on historical data.

A3.2 Stochastic dynamic programming

The stochastic dynamic programming model developed by Hertzler (2003, 2008) offers prospects to price water options in the presence of these violations of the conditions necessary for efficient trading. It is subject to fewer restrictions on its application in a water market than the other models described above, including the SKABS model that we use only to estimate option prices at the start of the contract period. Hertzler (2003, p. 52) explained how to write a call option contract on environmental resources, among other contracts, using water as an example. He showed that a stochastic differential equation could be used to model any probability distribution by transforming a Wiener increment, a crucial feature for success. General probability distributions could be
modelled by varying the two functions of the differential equation, making his model an improvement on previous models based on the generalised Wiener process. His model would need to be adapted to handle multiple-year options contracts, but Villinski (2003) demonstrated that this could be comfortably achieved, albeit with a model inferior to that proposed by Hertzler.